Reference:C:\Users\Bernhard Boser\Documents\Files\Lib\MathCAD\Default\defaults.mcd

L08: Practical Significance of Approximation

Even for simple circuits and using (linear!) small-signal analysis, exact equations are usually intractably complex. Complicated equations not only invite errors, but, more importantly, are very difficult to interprete. For example, it can be very challenging to determine from an equation with many terms how to modify the design of a circuit to meet all requirements such as gain and input & output resistance.

Approximations can be used to overcome these problems. Which approximations are valid of course depend on the circuit. Because of this, equations copied e.g. from textbooks can be used only after verification that all approximations made in the derivation are met.

Below we illustrate the use of approximations in the calculationn of the input resistance of a CE amplifier with degeneration.

R_{in} of CE with Degeneration

given

$$\frac{v_e}{R_E} + \frac{v_e - v_{in}}{r_\pi} + \frac{v_e - v_o}{r_o} - g_m \cdot (v_{in} - v_e) = 0$$

$$\frac{v_o}{R_C} + \frac{v_o - v_e}{r_o} + g_m \cdot (v_{in} - v_e) = 0$$

$$R_{in} = \frac{v_{in}}{\left(\frac{v_{in} - v_e}{r_\pi}\right)}$$

$$\text{find}(v_o, v_e, R_{in}) \rightarrow \begin{bmatrix} -v_{in} \cdot R_C \cdot \dfrac{(-R_E + g_m \cdot r_\pi \cdot r_o)}{(R_E \cdot r_\pi + r_\pi \cdot r_o + R_E \cdot r_o + r_\pi \cdot R_C + R_E \cdot R_C + g_m \cdot R_E \cdot r_\pi \cdot r_o)} \\ R_E \cdot v_{in} \cdot \dfrac{(R_C + g_m \cdot r_\pi \cdot r_o + r_o)}{(R_E \cdot r_\pi + r_\pi \cdot r_o + R_E \cdot r_o + r_\pi \cdot R_C + R_E \cdot R_C + g_m \cdot R_E \cdot r_\pi \cdot r_o)} \\ \dfrac{(R_E \cdot r_\pi + r_\pi \cdot r_o + R_E \cdot r_o + r_\pi \cdot R_C + R_E \cdot R_C + g_m \cdot R_E \cdot r_\pi \cdot r_o)}{(r_o + R_C + R_E)} \end{bmatrix}$$

Input resistance:

$$R_{in} = \frac{(1 + g_m \cdot R_E) \cdot r_\pi \cdot r_o + R_E \cdot (r_o + r_\pi + R_C) + r_\pi \cdot R_C}{r_o + R_C + R_E}$$

This equation is correct but unwieldy. It is difficult to see from the equation how R_{in} is affected by design parameters, how, for example, one could increase R_{in} to meet a specification.

Often approximations hold that permit significant simplifications. A few examples are presented below.

approx: $r_o >> R_E, R_C$

$$R_{in} = \left(1 + g_m \cdot R_E\right) \cdot r_\pi + R_E$$ or $$R_{in} = R_E \cdot \left(1 + g_m \cdot r_\pi\right) + r_\pi$$

approx: $(1+g_m R_E) >> R_C/R_E$

$$R_{in} = \left(1 + g_m \cdot R_E\right) \cdot r_\pi + R_E$$ or $$R_{in} = R_E \cdot \left(1 + \beta\right) + r_\pi$$

The two equations are equivalent. From the equations is apparent degeneration increases input resistance by $(1+g_m R_E)$ or $(1+\beta)$.

$$r_o = \frac{1}{g_o} = \frac{V_A}{I_C} \qquad \left(\frac{25\mu A}{100V}\right)^{-1} = 4M\Omega$$

Example: high gain amplifier with current source load